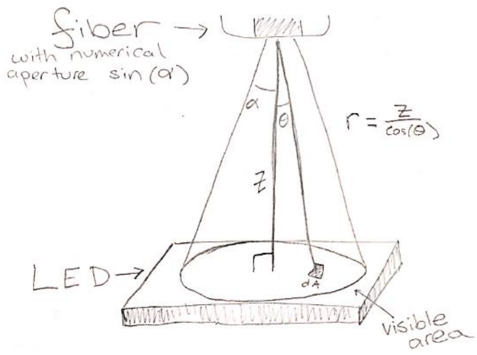
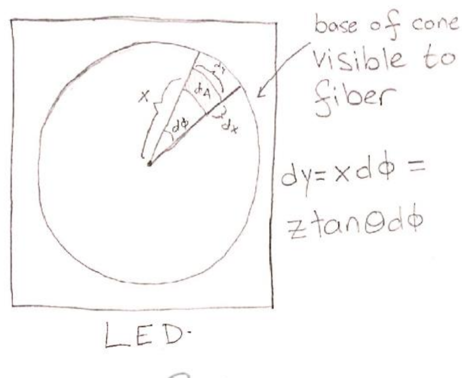
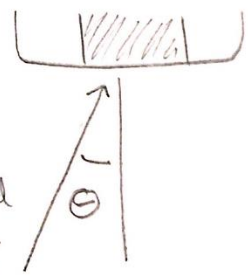
Light captured by a fiber is highest when the fiber is pressed directly against an LED, but at times it is practical to allow some distance between the fiber and the LED, either to allow for deformation or to allow for slight errors in alignment. If the distance between the fiber and the LED is small, change in distance has no effect on the total power captured.





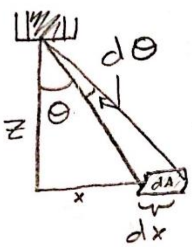
$dy = x\,d\phi = z\tan\theta\,d\phi$

P = power captured by fiber



incoming light hits fiber at angle θ. Power captured is related to perpendicular component $\cos\theta$

$$dP = \frac{1}{r^2} I(\theta)\cos(\theta)\,dA$$



$$dx = (\tan(\theta + d\theta) - \tan(\theta))z = z\,\frac{d\tan(\theta)}{d\theta}\,d\theta = z\sec^2(\theta)\,d\theta$$

$$dA = dy\,dx = z^2\tan(\theta)\sec^2(\theta)\,d\theta\,d\phi$$

$$dP = I(\theta)\frac{\cos^3(\theta)}{z^2}z^2\tan(\theta)\sec^2(\theta)\,d\theta\,d\phi$$

$$= I(\theta)\sin(\theta)\,d\theta\,d\phi$$

If the limits of integration for dθ and dφ have no z dependence, then the power received will be independent of distance, no matter the angular intensity of light emitted I(θ), which is a property of the LED. This is further shown by calculating the power collected from two different Luxeon Z LEDs which were used in testing our fibers.

Royal Blue Luxeon Z:

$$I(\theta) \simeq I_0 \cos(\theta)$$

where I_0 is the maximum perpendicular intensity.

$$P = \int_0^{2\pi} \int_0^{\alpha} I_0 \cos(\theta) \sin(\theta)\, d\theta\, d\phi =$$

$$\int_0^{\alpha} 2\pi I_0 \left(\tfrac{1}{2} \sin(2\theta)\right) d\theta =$$

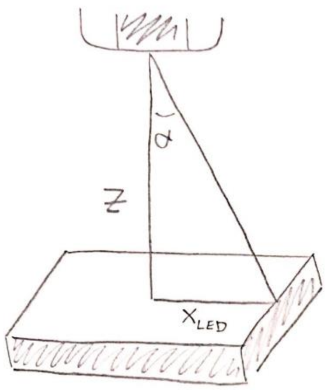
$$-I_0 \frac{2\pi}{4} \cos(2\theta)\Big|_0^{\alpha} = I_0 \frac{\pi}{2} (1 - \cos(\alpha))$$

Deep Red Luxeon Z:

$$I(\theta) = I_0 \left(-\left(\frac{\theta}{9.35}\right)^4 + \left(\frac{\theta}{280}\right)^3 - \left(\frac{\theta}{500}\right)^2 + 1\right)$$

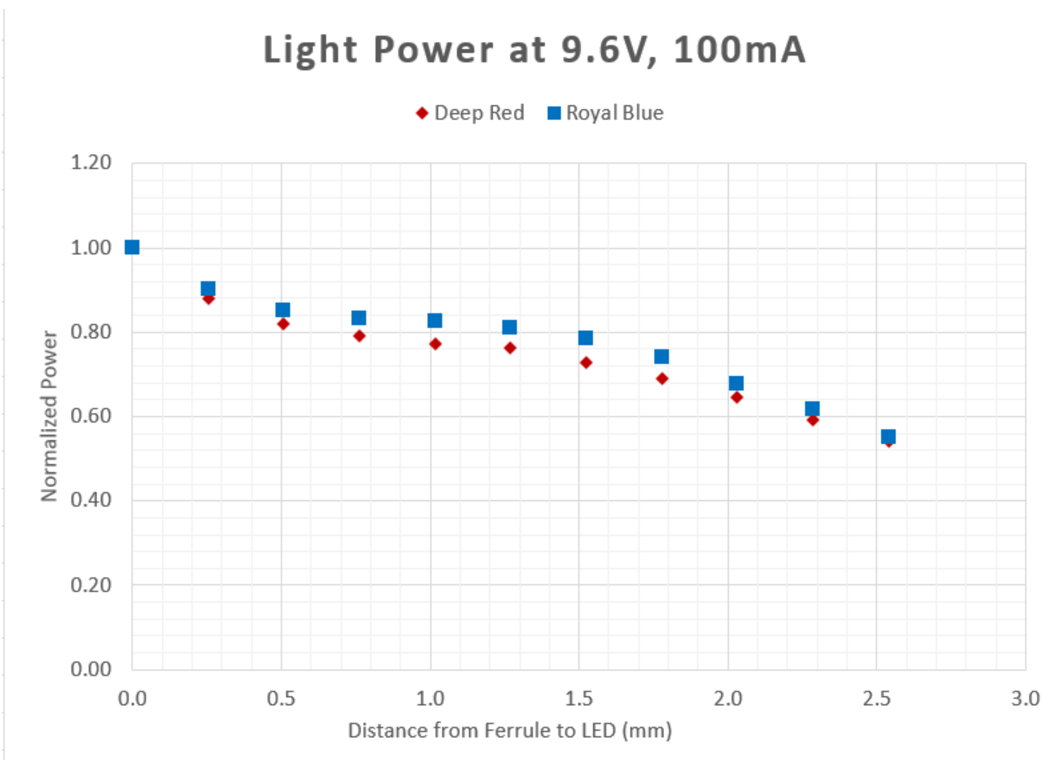
$$P = \int_0^{2\pi} \int_0^{\alpha} I_0 \left(-\left(\frac{\theta}{9.35}\right)^4 + \left(\frac{\theta}{280}\right)^3 - \left(\frac{\theta}{500}\right)^2 + 1\right) \sin\theta\, d\theta\, d\phi$$

$$= \int_0^{\alpha} 2\pi I_0 \left(\left(-\frac{\theta}{9.35}\right)^4 + \left(\frac{\theta}{280}\right)^3 - \left(\frac{\theta}{500}\right)^2 + 1\right) \sin\theta\, d\theta$$



If the perpendicular distance between the fiber and the LED is less than or equal to $z = x_{LED}/\tan(\alpha)$ where x_{LED} is the minimum distance between the center and the edge of the LED, the power captured will be constant. Up until this point, the LED fills the fiber's entire field of vision, so the $1/r^2$ decrease in intensity cancels out with the increase in visible area.

Our multi-mode fibers have a numerical aperture of .22, meaning light can enter from angles up to $\alpha = 12.7°$ or .222 radians. Luxeon Z LEDs are 1mm square, so the power captured by the fiber remains constant until the perpendicular distance between the LED and the fiber is greater than 2.21mm, assuming the fiber's line of sight is perfectly centered on the LED. Beyond this, the power captured will drop off significantly.



We found this to be true in our fiber tests. As shown above, percent of total power captured from each color stays high until around 2.2 mm, when the power captured begins to drop drastically. This is beneficial because it allows us to have some room for error and deformation in our setup.

It is worth noting that in the tests which produced the above result, alignment was done by sight only. Small errors in centering the fiber on the LED can cause power capture to drop off at much smaller distances.