The Effect of Dents in the BCAM Kinematic Mount

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May 2019

1 Background

When a BCAM is flexed on the mount, the mounting spheres will hit and press into the slot or flat. If it is flexed hard enough, the ball will create an indentation. We will briefly go through an explanation of the effects of a dent in the BCAM slot and flat on the axis.x and axis.y calibration constants.

2 Dent in the Slot

2.1 Dent depth

First, we must know the depth of the dent caused by the sphere. We do this by measuring the diameter of the dent, and deriving its depth from our knowledge of the diameter of the sphere. This can be found by starting with the right triangle formed from R, r_d and $R-h$ in Figure 1, where R is the radius of the sphere, r_d is the radius of the dent, and h is the depth of the dent.

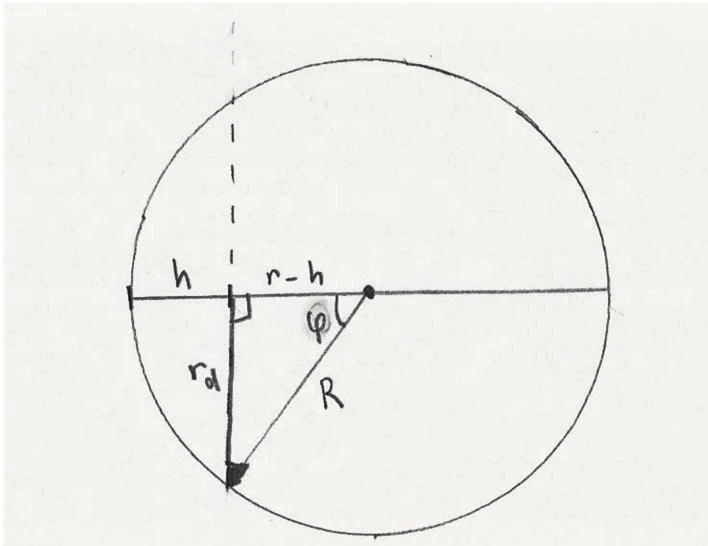

Figure 1: A drawing depicting the dent depth, h

Thus we have

$$r_d^2 + (R - h)^2 = R^2$$

$$r_d^2 + R^2 - 2Rh + h^2 = R^2$$

Because h is very small we can say

$$r_d^2 = 2Rh$$

and $r_d = \frac{d}{2}$, giving

$$h = \frac{d^2}{8R} \tag{1}$$

Where h is the depth of the dent, d is the diameter of the dent, and R is the radius of the sphere.

2.2 Rotation Due to a Dent in the Slot

We will assume that when the BCAM is remounted, the center of the mounting sphere in the slot will now be shifted by this distance h. We will consider a the rotation about the y-axis separately from the rotation about the z-axis. For the purposes of this derivation we will study rotation about the y-axis, thus, we know that only the component of h in the x-z-plane is relative.

Since we know that h is perpendicular to the slot which is at a 45^o angle to the y axis, we know the component that lies in the x-z plane (and that is relevant to rotation about the y axis) is

$$h_t = h\sin(45^o) = \frac{h}{\sqrt{2}} \tag{2}$$

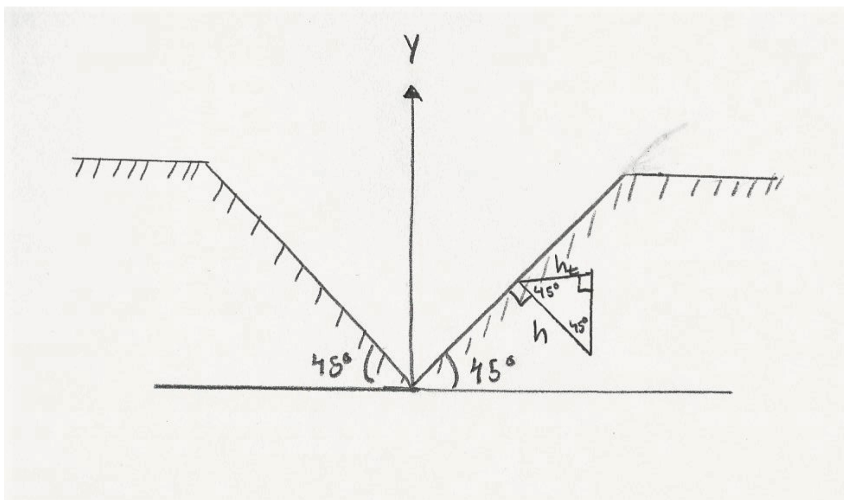

Figure 2: Cross section of the slot, decomposing h to the relevant component. The horizontal axis is a line that lies in the xz-plane

Using this, we can compute the amount that the entire BCAM will rotate about the y-axis. For small angles we know

$$\sin(\theta) \approx \theta \tag{3}$$

We can use this approximation because, as shown in Figure 3, $h_t << l$. Thus

$$\theta = \frac{h_t}{l} \tag{4}$$

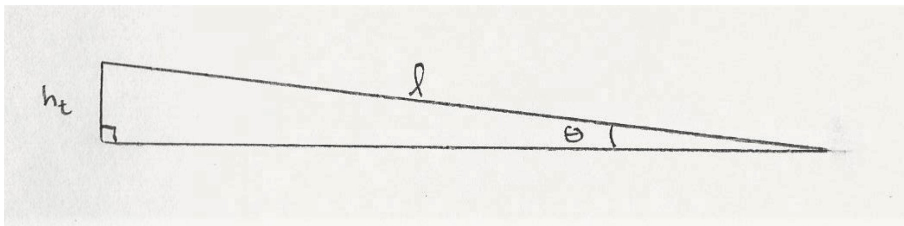

Figure 3: Drawing of angle θ, and how it is derived from h_t and l.

Where l is the distance from the center of the cone to the slot and θ is the rotation about in degrees. So, by combining equations 1, 2, and 4, we find that θ is

$$\theta = \frac{d^2}{8\sqrt{2}Rl} \tag{5}$$

Since the BCAM is a rigid body, this θ is also the change in the calibration constant axis.x and axis.y.

2.3 Case Study

When originally performing a flex test on a Blue D-BCAM, it was found that this caused dents in the slot. We used a Celestron Digital Microscope and the Celestron capture software to view, and take images of the dents in the slot, as seen below.

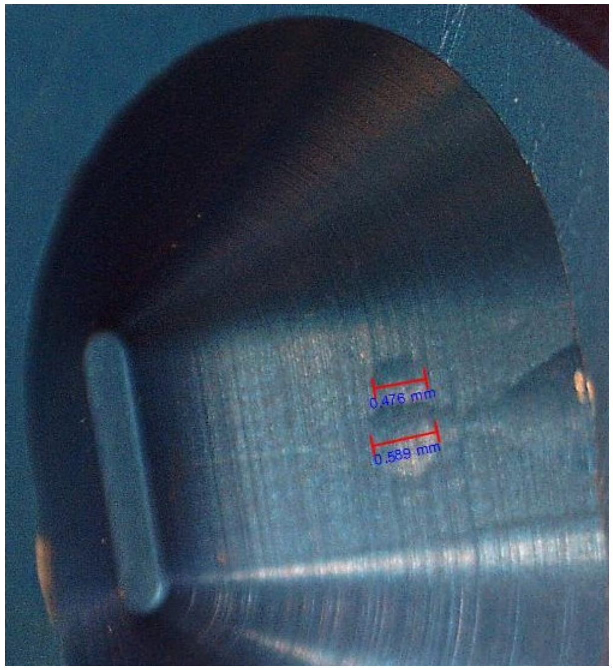

Figure 4: Image of Dents in a Blue DBCAM chassis

We took three separate images, calibrated the measurement tool to the length of the slot and used the Celestron measurement program to measured the diameter of the dent. We found it to be $d = 0.598 \pm 0.024$mm. The other values calculated to find theta are listed in the table below.

d	R	l	θ
0.598 $\pm$ 0.024mm	6.35mm$\pm$ 1μ m	75.95$\pm$0.025 mm	66 $\pm$ 5μ rad

As you can see in the table above, the change in the x-axis $\theta = 66\mu$rad. Since our CCD calibration for the x and y axis calibration constant is accurate to $50\mu rad$, this is something we suspect we will be able to see.

To test if we can in fact observe a change in the x and y axis calibration constants we compared the calibration constants for the DBCAM obtained before it was flexed with those taken after. On the table below we list the change we observed in the axis calibration constants.

Change in Axis Measurements(μrad)			
x-axis front	y-axis front	x-axis back	y-axis back
1.0	13	16	20

These values are smaller than what we predicted, and they are also smaller than the accuracy of the BCAM calibration. Thus we can not conclude that there has been any significant change in the calibration constants of this DBCAM.

However, as you can see in Figure 4, there are actually two dents of different sizes in the BCAM slot. Thus it difficult for us to state which, if any, dent the mounting sphere is sitting in. If it was actually resting in the smaller slot, which has a diameter of 0.486±0.039mm, we would predict a change of 43 $\pm 7\mu$rad in the axis.x and axis.y calibration constants, which is insignificant as it is less than 50μrad. Thus, while it is possible that dents in the slot of the BCAM over 0.59mm in diameter will cause a detectable change in the axis.x and axis.y calibration constants, we did not observe that in this BCAM.

3 Dent in the Flat

3.1 Rotation Due to a Dent in the Flat

As with the slot, we will start our analysis of dents in the flat with the dent depth given by Equation 1, $h = \frac{d^2}{8R}$. Because h will be in the y-direction, the rotation of the BCAM will be about the z-axis. We will then also make the assumption that the slot is still in the same location in relation to its mounting sphere. Thus, we can say, again using the approximation $\sin\theta \approx \theta$, for small angles, we have

$$\theta = \frac{h}{s} = \frac{d^2}{8Rs} \tag{6}$$

Where s is the distance from the center of the flat to the center of the slot as defined by the position off the mounting spheres. For the BCAM mount this distance is $s = 41.96$mm. Thus the change in the measurement of axis.y of a BCAM will be

$$\theta = \frac{d^2}{8R41.96} \tag{7}$$

4 Appendix: Sample Calculations and list of Constants

4.1 Constants

When mounting a BCAM, we used high precision quarter inch mounting spheres. The diameter in mm, as well as the constants of l and s used in the formulae for the change in the axis measurements are listed in the table below.

Constants for a BCAM		
Diameter of the Mounting Sphere (R)	Distance from the slot to the cone (l)	Distance from the flat to the slot (s)
6.35mm$\pm$ 1μ m	75.95$\pm$ 0.025 mm	41.96 $\pm$ 0.025 mm

These constants are for all BCAMs.

4.2 Sample Calculations

4.2.1 Depth of Dent in Flat

For example, say that you notice a dent on the Flat of a Blue N-BCAM, that you measure to be 0.600mm in diameter. Thus, the depth of the dent will be

$$h = \frac{d^2}{8R} = 7.09 \times 10^{-3} mm$$

4.3 Change in axis.y

Once we know the depth of the dent, we can find the change in the axis.y calibration constant using equation 7.

$$\theta = \frac{h}{s} = \frac{7.09 \times 10^{-3}}{41.96} = 16.9 \mu rad$$

This change is less than 50μrad, thus it would not be significantly detectable with the BCAM calibrator.